Mathematical Physics

Individual Overall Contest

To complete this assessment, answer at least 1 of the following 2 problems

Problem 1

Statistical Mechanics: Feynman gas

We have a one-dimensional classical gas consisting of N impenetrable particles (i.e. the particles cannot cross each other) which move in the segment $[0, L] \subset \mathbf{R}$. The particles screen each other, so that each particle interacts only with the particles next to it along the line. The Hamiltonian is

$$H_N = \sum_{i=1}^{N} \frac{p_i^2}{2m} - \lambda \sum_{i=1}^{N-1} \log |x_{i+1} - x_i|$$

Compute the Gibbs free energy in the thermodynamic limit ($N \to \infty$ with L/N fixed).

Problem 2

Relativistic Mechanics A particle with mass m and electric charge e is launched horizontally with velocity v from a height h above the ground. It then falls under the influence of constant vertical electric field $\vec{E}$ and lands a distance R away. Find the relativistic Hamiltonian, the Hamiltonian equations of motion, and express R in terms of h, m, and v. (Set $c = 1$ for simplicity and neglect air resistance and gravity.)